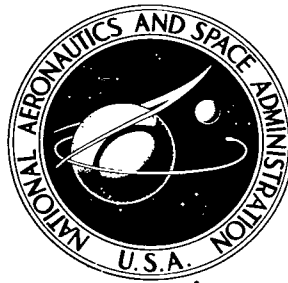


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SIMPLE EQUATIONS FOR CALCULATING TEMPERATURE DISTRIBUTIONS IN RADIATING GRAY GASES

by Robert G. Ragsdale and Albert F. Kascak

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Cleveland, Ohio



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ABSTRACT

Algebraic equations are presented for the temperature distribution in a radiating gray gas. The equations are in good agreement with exact solutions for the entire range of opacities for a constant absorption coefficient and uniformly distributed heat sources, for spherical, cylindrical, or slab geometries. It is shown numerically how to include a variable absorption coefficient and a nonuniform heat source distribution.

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SUMMARY

Equations are derived and presented for the temperature distribution throughout a heat generating, radiating gray gas. The results are simple, algebraic equations that require only a slide rule for use. The results are in good agreement with exact answers over the entire range of opacities, from zero to infinity, for any heat flux, for a constant absorption coefficient and a uniform distribution of volumetric heat sources. Equations are given for spherical, cylindrical, and slab geometries. The equations go to the correct form in the limiting case of a transparent gas (zero opacity), and they go to the diffusion theory limit for an opaque gas (infinite opacity). At intermediate opacities ($0.1 \leq \text{optical thickness} \leq 10$), the equations give temperatures that are within 3 percent of a numerical solution to the exact transport equation.

These equations are based on a new model of the radiative process occurring very near the outer edge of the gas. The equation for the edge temperature is

$$\frac{T_e}{T_b} = \left[\frac{1}{2} \left(1 + \frac{a}{\tau} \right) \right]^{1/4}$$

where T_b is the brightness temperature, defined so that σT_b^4 gives the radiated heat flux, τ is the optical thickness, kL for a slab of thickness L or kD for a sphere or cylinder of diameter D . The constant a is 1, 2, or 3, for a slab, cylinder, or sphere, respectively.

For a constant absorption coefficient and uniformly distributed heat sources, the temperature distribution in the gas is given by

$$\frac{T}{T_b} = \left[\frac{1}{2} \left(1 + \frac{a}{\tau} \right) + \frac{3\tau}{16} \left(1 - \bar{x}^2 \right) \right]^{1/4}$$

where a and τ are defined as above, and \bar{x} is the dimensionless distance from the center of the gas ($\bar{x} = 0$) to the edge ($\bar{x} = 1$).

It is also shown numerically how to include a temperature-dependent absorption coefficient and a nonuniform heat source distribution. These equations are presented, but not compared to exact answers.

INTRODUCTION

Numerous research papers and textbooks (e.g., refs. 1 and 2) have been published on the subject of radiative heat transfer. Problems in which gases participate are probably the most complicated because gas absorption coefficients are dependent on both the radiation wavelength and the gas temperature. Generally, either approximations or numerical computer solutions, or both, are required to obtain quantitative answers to specific problems. It is the purpose of this report to present some simple equations that require only a slide rule to obtain temperature distributions within radiating gases.

In order to attack this problem, it is necessary to assume that the gas is gray. That means that the absorption coefficient is the same for all wavelengths of radiation. Although this is not generally true of real gases, this approach does yield some engineering-estimate tools and, perhaps, some insight into radiative processes that would not otherwise be obtainable. It may be possible to account for spectral behavior by using a gray gas analysis along with some average absorption coefficient (refs. 3 and 4).

In this report, the equations for temperature distribution throughout a heat generating, radiating gas are obtained first for the case where the absorption coefficient is not a function of temperature, and where the volumetric heat sources are distributed uniformly throughout the gas volume. Then, these equations are rewritten for the more general case where the absorption coefficient is an arbitrary function of temperature and the heat sources are distributed as some arbitrary function of position in the gas volume. Although this general case obviously represents a considerably more complicated physical situation, it does not add much more numerical difficulty.

Figure 1 shows a model of this problem for the three geometries investigated, a sphere, a cylinder, and a slab. The variables of the problem are the physical size and shape of the gas volume, the value of the absorption coefficient, and the amount of power generated within the gas.

A brief review of some publications on gaseous radiant heat transfer shows how this work fits into the structure of what has gone before. Even with the gray gas assumption, numerical methods are required to obtain solutions to the exact transport equations (refs. 5 to 9). For optically dense situations, such as in stars, approximations have been developed to give the temperature distribution near the surface (refs. 10 and 11).

The most straightforward situation is that of a relatively transparent gas. In this case, the radiation emitted by every unit volume escapes the gas region with no appreciable absorption by any of the intervening volumes. The gas is therefore at constant temperature throughout. The radiated heat flux and the gas temperature are related by a simple expression that contains only the absorption coefficient, as long as the gas is in local thermodynamic equilibrium.

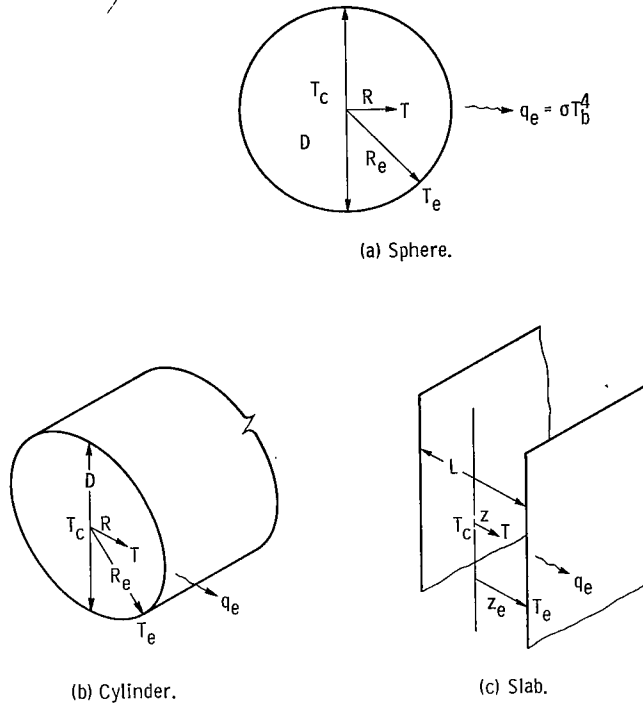


Figure 1. - Model of radiating gas.

Highly absorbing, or opaque, gases present a much more difficult situation, even when the absorption coefficient is constant. This kind of problem is generally attacked by using a diffusion approximation to the radiative transport equation. This procedure works quite successfully, at least in the interior regions. The diffusion equation does not work at the boundary of a region, however, because the heat flow is not a diffusive process within the last one or two photon mean free paths of the outer edge.

Thus, although the diffusion approximation works in the interior of opaque gases, the problem of the edge temperature remains. Chandrasekhar has published an approximation that works just in the outer regions of an opaque gas (ref. 11). But his method does not work for gases of intermediate opacity, or for transparent gases. Usiskin and Sparrow have presented some computer solutions for flat plates (ref. 8) and spheres (ref. 9), but only for intermediate opacities from 0.1 to 2. Their results do not provide simple algebraic equations that can be used for other opacities.

Deissler (ref. 12) attacked the problem by considering the fact that there would be an energy "jump" between the gas and its surroundings if the heat transfer were solely by radiation. He presents second-order differential equations for this energy jump for spherical, cylindrical, and flat plate geometries. He shows that the jump boundary conditions extend the range of validity of the diffusion approximation to comparatively low values of optical thickness. His results are presented in terms of heat transfer between

walls or from walls to flowing gases. No algebraic equations for the temperature distributions throughout a heat generating gas are given. There is also no detailed examination of just how far into the transparent gas regime, the jump boundary conditions apply.

It is the purpose of this study to do these latter two things: First, simple algebraic equations are obtained that give the temperature distribution throughout a radiating gas as a function of absorption coefficient, power generation, and geometry. New expressions are derived for the edge temperature. The results obtained are compared with the results of Deissler and Chandrasekhar. A computer program was also written and is used to obtain exact solutions over the range of opacity from 0.01 to 1000. The results from the simple equations are compared with these computer answers. The equations work for all opacities, from zero to infinity, for all power generation rates, and for spheres, cylinders, and slabs. The temperature distribution throughout the entire gas, from center to edge, can be easily calculated with a slide rule.

The equations are first obtained for a constant absorption coefficient and uniform heat sources. They are then rewritten for the more general case where the absorption coefficient is a function of temperature. The form $k = cT^n$ is used herein to illustrate the procedure. Practically any other function could be used with little difficulty. It is also shown how to incorporate any arbitrary distribution of heat sources into the equations. This is done by calculating the temperature distribution in an induction-heated uranium plasma. For this case the absorption coefficient varies with temperature, and heat generation varies with radial position.

SYMBOLS

A	cross-sectional area
a	constant in eq. (22)
c, c_1, c_2	mathematical constants
D	diameter
e	gas emissive power, σT^4
f	kernel of integral equation
g, h	functions used to put differential equation into standard form
k	linear absorption coefficient
L	slab thickness
l	path length of beam of radiation
M	molecular weight

m	mass of gas in a unit volume
\vec{n}	unit vector normal to surface
P	function use to put differential equation into standard form
Q	heat generation rate per unit volume
q	heat flux, heat flow per unit time per unit area
R	radius
\vec{R}	unit vector in R direction
\bar{R}	radius normalized to edge radius, R/R_e
\mathcal{R}	gas law constant
S	surface area
T	temperature
V	volume
z	coordinate normal to slab surface, zero at slab midplane
\bar{z}	normalized slab coordinate, z/z_e
β	beta function
δ	skin depth
ϵ	sum of higher-order terms in Taylor series
η	nondimensional heat flux
τ	opacity (or optical dimension), kD or kL
τ^*	optical depth into gas from edge
ξ	normalized emissive power
ρ	gas density
σ	Stefan-Boltzmann constant
χ	general coordinate, either R or z
$\bar{\chi}$	dimensionless general coordinate, either \bar{R} or \bar{z}

Subscripts:

av	average
b	brightness
c	center

e	edge
f	field
g	gas
s	source
w	wall
x	argument of beta function, see eq. (36)

Superscript:

n	temperature exponent, see eq. (27)
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ANALYSIS

In this section equations are derived that give the gas temperature as a function of position within the gas volume, absorption coefficient, and amount of heat being generated within the gas. There is one such equation for each of the three geometries considered - sphere, cylinder, and slab.

First, an equation is derived for the temperature distribution normalized to the "edge" temperature. This is done for a constant absorption coefficient and uniform heat generation. Next, an expression for the edge temperature is derived. These two equations give the desired equation for gas temperature from the center out to, and including, the edge temperature. Then, this equation is rewritten for the case where the absorption coefficient is an arbitrary function of temperature. Next, it is shown how to easily write similar equations for any specified variation of heat sources throughout the gas. Finally, a numerical computer solution to the transport equations is formulated. The computer solution was used to obtain answers from the exact radiant transport equation in order to check the accuracy of the diffusion equations obtained. Expressions similar to the ones derived herein are also obtained from the differential equations presented in reference 12. A comparison of these two sets of equations is presented and discussed in this report.

Diffusion Equations

The diffusion equation for radiative heat flow is derived first for a parallel-plate, or slab, geometry. This derivation illustrates the mathematics. Then, the final equations for cylindrical and spherical geometries are given without derivation because the steps are analogous.

The gas to be analyzed is contained in a volume between two parallel planes that are some distance L apart. There is steady-state, so that the temperature at a given point within the volume is unchanging with time. The energy that is radiated is produced by heat sources uniformly distributed throughout the gas volume. In real situations, such heat sources could be due to nuclear fusion, nuclear fission, electric currents, or chemical combustion.

In order to obtain an equation relating gas temperature and heat flux, it is assumed herein that the heat flow is a diffusion process; that is, the local flux is proportional to a local gradient:

$$\text{Flux} = \text{Coefficient} \times \text{Gradient} \quad (1)$$

For radiative heat transfer in a gas, equation (1) is written as

$$q = -\frac{4}{3k} \frac{\partial e_g}{\partial z} \quad (2)$$

where q is the heat flux in the $+z$ direction, k is the absorption coefficient, and e_g is σT^4 , or the gas emissive power. Equation (2) has been derived by a number of investigators, beginning with Rosseland in 1931. A recent derivation is given by Deissler in reference 12.

For a parallel-plate geometry, the heat flux is related to the volumetric heat generation rate by

$$q = Qz \quad (3)$$

In this first derivation, Q is taken to be constant throughout the gas volume. Equation (2) can be written as

$$\frac{\partial e_g}{\partial z} = \frac{-3kQ}{4} z \quad (4)$$

Integrating equation (4) from some general location within the gas z to the outer edge z_e and substituting σT^4 for e_g give

$$\sigma T^4 - \sigma T_e^4 = \frac{-3kQ}{8} (z^2 - z_e^2) \quad (5)$$

Here z is zero at the center plane, as shown in figure 1(c). This can be generalized somewhat, since the edge heat flux is

$$q_e = Qz_e \quad (6)$$

and the optical thickness of the slab of gas is

$$\tau \equiv kL = 2kz_e \quad (7)$$

Equations (5), (6), and (7) give

$$\sigma T^4 = \sigma T_e^4 + \left(\frac{3\tau}{16}\right)q_e (1 - z^2) \quad (8)$$

where z is z/z_e , and varies from 0 at the center of the gaseous slab to 1 at either of the bounding surface planes. The total distance between the two surfaces is $2z_e$, or L .

Equation (8) is instructive because it begins to expose some of the characteristics of a heat generating, radiating gas. For example, the difference between the center temperature and the edge temperature is determined by the product of the opacity and the edge heat flux, independent of the absolute size or the geometrical shape for the three geometries considered in this analysis.

However, equation (8) cannot be used to obtain actual temperatures because the edge temperature is unknown. It can be arbitrarily specified as a boundary condition, as was done in reference 13. Deissler obtained an expression for the edge temperature from a second-order diffusion equation. A different equation for the edge temperature is obtained in the following section from a new model of the radiative processes occurring at the edge of the gas.

Edge Temperature

An equation for the edge temperature of a radiating volume of gas is obtained by writing a heat balance on the outermost "layer" of gas. The layer considered is of thickness Δz , where Δz is small enough that $k\Delta z$, or $\Delta\tau$, is much less than 1; that is, the layer is optically thin. Therefore, all of the gas in this layer is at the same temperature, T_e . Further, the heat flux emitted in this layer is given by

$$q_{\text{emit}} = 4k\sigma T_e^4 \Delta z \quad (9)$$

Now, the heat energy emitted in this layer must be exactly equal to the heat generated in the layer, plus the amount absorbed in it. Some energy is absorbed, since all of the energy generated within the gas volume must pass through the layer in order to escape. The balance, in words, is

$$\text{Emitted} = \text{Absorbed} + \text{Generated} \quad (10)$$

The generated heat is simply

$$q_{\text{gen}} = Q \Delta z \quad (11)$$

The heat flux entering the layer is q_e . The intensity, I , associated with this flux is attenuated along any given ray according to the law:

$$I = I_e e^{-kl} \quad (12)$$

When all rays are summed up, the average, or ''mean'', beam length across a slab is just $2\Delta z$, in the limiting case of Δz approaching zero, (ref. 14). Thus,

$$q = q_e e^{-2k \Delta z} \quad (13)$$

Finally, since $e^{-x} = 1 - x$ for $x \ll 1$, and since the fraction absorbed is 1 minus the fraction transmitted, the absorption in the outer layer is given by

$$q_{\text{abs}} = 2q_e k \Delta z \quad (14)$$

Equations (9), (11), and (14) are substituted into the energy balance equation (10)

$$4k\sigma T_e^4 \Delta z = 2q_e k \Delta z + Q \Delta z \quad (15)$$

Dividing by $4k \Delta z$ and noting that $\tau \equiv 2kz_e$ and that $q_e = Qz_e$ we get

$$\sigma T_e^4 = \frac{q_e}{2} \left(1 + \frac{1}{\tau} \right) \quad (16)$$

This equation is interesting in itself, because it shows that the edge temperature is determined uniquely by the edge heat flux leaving the surface and the optical thickness of the slab τ . It further shows that, for very opaque gases (where $1/\tau \ll 1$), the edge temperature is determined solely by the radiated heat flux. The radiated heat flux is twice the amount that would be radiated by a blackbody at the edge temperature T_e . This agrees with estimates of stellar photosphere temperatures (ref. 10).

Equation (16) gives the edge temperature for a slab of gas. For a cylinder, an analogous development exists, except that $q_e = QR_e/2$. This leads to

$$\sigma T_e^4 = \frac{q_e}{2} \left(1 + \frac{2}{\tau}\right) \quad (17)$$

Similarly, for a sphere $q_e = QR_e/3$ and

$$\sigma T_e^4 = \frac{q_e}{2} \left(1 + \frac{3}{\tau}\right) \quad (18)$$

Thus, the general formula for the edge temperature is

$$\sigma T_e^4 = \frac{q_e}{2} \left(1 + \frac{a}{\tau}\right) \quad (19)$$

where $a = 1$ for a slab, $a = 2$ for a cylinder, and $a = 3$ for a sphere.

General Temperature Distribution

Equation (8) gave the temperature distribution throughout the gas relative to the edge temperature T_e . Equation (19) gives the edge temperature. These two equations combine to give

$$\sigma T^4 = \frac{q_e}{2} \left[1 + \frac{a}{\tau} + \frac{3\tau}{8} \left(1 - \bar{\chi}^2\right)\right] \quad (20)$$

Where $\bar{\chi}$ is \bar{z} for a slab and \bar{R} for a cylinder or a sphere. Now, let a brightness temperature T_b be defined so that σT_b^4 gives the radiated heat flux q_e :

$$q_e = \sigma T_b^4 \quad (21)$$

Then, equation (20) becomes

$$\frac{T}{T_b} = \left[\frac{1}{2} \left(1 + \frac{a}{\tau} \right) + \frac{3\tau}{16} \left(1 - \bar{\chi}^2 \right) \right]^{1/4} \quad (22)$$

Equation (22) is the desired goal. It is a simple, algebraic equation for the temperature distribution throughout the gas from the center ($\bar{\chi} = 0$) to the edge ($\bar{\chi} = 1$). It does give the established temperature distribution for the two limits of an opaque gas ($\tau \rightarrow \infty$) and a transparent gas ($\tau \rightarrow 0$). For an opaque gas,

$$q_e = 2\sigma T_e^4 \quad (23)$$

and for a transparent gas,

$$q_e = 2\tau\sigma T_e^4 \quad (24)$$

$$q_e = \tau\sigma T_e^4 \quad (25)$$

$$q_e = \frac{2\tau}{3} \sigma T_e^4 \quad (26)$$

for a slab, cylinder, and a sphere, respectively.

Equation (22) was developed for a constant absorption coefficient. With little more trouble, the absorption coefficient can be an arbitrary function of temperature. To illustrate this, equation (22) is rewritten for the case where $k = cT^n$ in the following section.

Temperature-Dependent Absorption Coefficient

The general diffusion equation for a constant volumetric heat source distribution was

$$\frac{\partial q_e}{\partial z} = \frac{-3Q}{4} kz \quad (4)$$

Now, if $k = cT^n$, equation (4) is

$$\int_T^{T_e} \frac{\sigma dT^4}{cT^n} = \frac{-3Q}{4} \int_z^{z_e} z dz \quad (27)$$

This equation can be integrated to give the equivalent of equation (5) for a constant absorption coefficient:

$$\sigma T^4 \left(\frac{T_e}{T} \right)^n = \sigma T_e^4 + \left(\frac{4-n}{4} \right) \left(\frac{3\tau_e}{8} \right) \left(\frac{q_e}{2} \right) (1 - \bar{z}^2) \quad (28)$$

where τ_e is given by $2z_e k_e$. Similarly, an edge temperature equation can be developed analogously to equation (16):

$$\sigma T_e^4 = \frac{q_e}{2} \left(1 + \frac{1}{\tau_e} \right) \quad (29)$$

Combining equations (28) and (29) gives

$$\frac{T}{T_b} = \frac{\left[\frac{1}{2} \left[1 + \frac{1}{\tau_e} + \left(\frac{4-n}{4} \right) \left(\frac{3\tau_e}{8} \right) (1 - \bar{z}^2) \right] \right]^{1/4-n}}{\left[\frac{1}{2} \left(1 + \frac{1}{\tau_e} \right) \right]^{n/4}} \quad (30)$$

Equation (30) is a little more complicated than its constant absorption coefficient counterpart (eq. (22)) because of τ_e . The term τ_e depends T_e because

$$\tau_e = 2z_e k_e = 2z_e c T_e^n \quad (31)$$

The values of τ_e and T_e have to be determined by an iteration between equations (29) and (31). Thus, equation (30) gives the gas temperature T as a function of position z .

Although the algebra is not carried out here, it should be apparent that virtually any distribution of heat sources could be accommodated in equation (4). The integration that has been carried out for constant Q is

$$\int \frac{de_g}{k} = -\frac{3Q}{4} \int z dz \quad (32)$$

No additional complexity is introduced if Q is some function of z . Thus, any $Q(z)$ can be handled analytically so long as the integration $\int Q(z)z dz$ can be carried out.

Average Temperature

An average gas temperature can be calculated from the equation for the local temperature distribution. However, it turns out to be impossible to obtain a closed-form solution for average temperature in the case of a sphere or a slab. For these two cases, the average temperature is given in terms of an integral function which is then evaluated by numerical integration. For a cylinder, the required integration is carried out to obtain a closed-form solution.

It is first necessary to define an average temperature. The average temperature used herein is a "mixed mean cup" temperature; that is, obtained by weighting the local temperature with the local density. The mass of gas in a unit volume (of cross-sectional area A and unit thickness) of gas between two parallel plates is

$$m = \rho_{av} V = \frac{P}{RT_{av}} A \quad (33)$$

This mass is also given by integrating the local density over the volume

$$m = \int_0^1 \frac{PA d\bar{z}}{RT(\bar{z})} \quad (34)$$

Equating equations (33) and (34) gives the defining equation for the average temperature, normalized to the brightness temperature T_b :

$$\frac{T_{av}}{T_b} = \frac{1}{\int_0^1 \frac{d\bar{z}}{T(\bar{z})/T_b}} \quad (35)$$

where $T(\bar{z})$ is the local temperature. It is given by equation (22) with $\bar{\chi} = \bar{z}$ as a function of the opacity τ and the local position \bar{z} . Equation (22) is substituted into equation (35) to obtain an equation for the average temperature.

For a slab geometry, the following expression was obtained:

$$\frac{T_{av}}{T_b} = \frac{\frac{T_c}{T_b}}{\frac{\beta_{\chi}\left(\frac{1}{2}, \frac{3}{4}\right)}{2\sqrt{\chi}}} \quad (36)$$

The term $\beta_{\chi}(1/2, 3/4)$ is an incomplete beta function. It cannot be analytically integrated. The argument χ is given by

$$\chi = \frac{\frac{3\tau}{\tau}}{1 + \frac{1}{\tau} + \frac{3\tau}{8}} \quad (37)$$

The expression for $\beta_{\chi}(1/2, 3/4)$ is

$$\beta_{\chi}\left(\frac{1}{2}, \frac{3}{4}\right) = \int_0^{\chi} t^{(1/2)-1} (1-t)^{(3/4)-1} dt \quad (38)$$

As τ varies from zero to infinity, χ varies from zero to 1. Equation (38) was numerically integrated, and the value of the integral is shown in figure 2.

In a similar fashion, for a sphere the equation for average temperature is

$$\frac{T_{av}}{T_b} = \frac{\frac{T_c}{T_b}}{\frac{\beta_{\chi}\left(\frac{3}{2}, \frac{3}{4}\right)}{\frac{2}{3}\chi^{3/2}}} \quad (39)$$

where here β_{χ} is given by

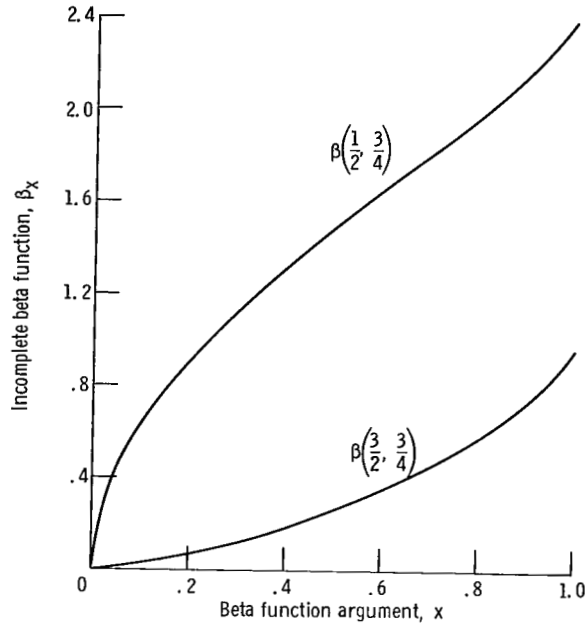


Figure 2. - Incomplete beta functions;

$$\beta_x(n, m) = \int_0^x t^{n-1} (1-t)^{m-1} dt.$$

$$\chi = \frac{\frac{3\tau}{8}}{1 + \frac{3}{\tau} + \frac{3\tau}{8}} \quad (40)$$

The opacity τ is based on the sphere diameter.

For a cylinder, the same approach gives an expression that is integrable, so an analytic expression was obtained. The average temperature is given by

$$\frac{T_{av}}{T_b} = \frac{\left(\frac{3}{4}\right)\left(\frac{3\tau}{16}\right)}{\left(\frac{1}{2} + \frac{1}{\tau} + \frac{3\tau}{16}\right)^{3/4} - \left(\frac{1}{2} + \frac{1}{\tau}\right)^{3/4}} \quad (41)$$

Equations (36), (39), and (41) could be rewritten in terms of T_e and T_c rather than τ . For example, equation (41) can be reformulated using the following relations that come from equation (22) for a cylinder ($a = 2$):

$$\frac{T_e^4}{T_b^4} = \frac{1}{2} + \frac{1}{\tau}$$

$$\frac{T_c^4}{T_b^4} = \frac{1}{2} + \frac{1}{\tau} + \frac{3\tau}{16}$$

$$\frac{T_c^4 - T_e^4}{T_b^4} = \frac{3\tau}{16}$$

These three equations can be substituted into equation (41) to give

$$T_{av} = \frac{\frac{3}{4} (T_c^4 - T_e^4)}{T_c^3 - T_e^3} \quad (42)$$

Similar algebraic expressions cannot be obtained for a slab or a sphere because of the beta functions. However, the argument χ in equations (37) and (40) can be written as

$$\chi = 1 - \left(\frac{T_e}{T_c} \right)^4 \quad (43)$$

Jump Boundary Condition Equations

An equation very similar to equation (22) for temperature distribution can be obtained from the differential equations presented by Deissler in reference 12. The procedure is illustrated here for a slab geometry, and then the equations for a cylinder and a sphere are written without derivation.

For a slab of radiating gray gas, Deissler's equation (17) gives the difference between the emissive power of the edge gas and an adjacent wall as

$$e_{g,e} - e_b = \left(\frac{1}{\epsilon} - \frac{1}{2} \right) q_e - \frac{1}{2k^2} \frac{\partial^2 e_g}{\partial z^2} \quad (44)$$

For the case considered here, there is no wall, so $e_b = 0$ and $1/\epsilon = 1$. Therefore,

$$e_{g,e} = \frac{q_e}{2} - \frac{1}{2k^2} \frac{\partial^2 e_g}{\partial \xi^2} \quad (45)$$

The second derivative of emissive power is obtained from

$$\frac{\partial e_g}{\partial z} = -\frac{3k}{4} Qz$$

as

$$\frac{\partial^2 e_g}{\partial z^2} = -\frac{3kQ}{4}$$

Substituting this expression into equation (45) and using $q_e = Qz_e$ give

$$e_{g,e} = \sigma T_e^4 = \frac{q_e}{2} \left(1 + \frac{3}{2\tau} \right) \quad (46)$$

where τ is $2kz_e$.

This expression can be compared with equation (19) of this report. The two equations are similar in form, but the equation based on the jump boundary condition (eq. (46)) gives a constant $3/2$, where the equation of this report (eq. (19)) gives 1. These two equations are compared, and the significance of the different value of the constant discussed is in RESULTS AND DISCUSSION. For an opaque gas ($\tau \rightarrow \infty$), both equations give the correct diffusion limit:

$$\sigma T_e^4 = \frac{q_e}{2}$$

A similar development for a sphere and for a cylinder, using jump boundary conditions, leads to the following general relation:

$$\sigma T_e^4 = \frac{q_e}{2} \left(1 + \frac{a}{\tau} \right) \quad (47)$$

TABLE I. - CONSTANTS FOR EQUATIONS
(19), (22), AND (47)

Geometry	This report	Jump boundary condition
	Constant, a	
Sphere	3	3
Cylinder	2	2.25
Slab	1	1.5

where $a = 1.5, 2.25$, or 3 for a slab, cylinder, or sphere (see table I).

Numerical Solution of Transport Equation

The object of this numerical analysis was to obtain "exact" answers that could be used to check the accuracy of the simple, approximate equations presented in this report. It is assumed that only one coordinate is necessary to describe the system. It is further assumed that the heat flux is known and is symmetric about the center. The analysis considers a gas with constant absorption, and in local thermodynamic equilibrium.

The heat flux equation can be derived from reference 11 (ch. 1) by substituting equation (52) of reference 11 into equation (7) of reference 11 and converting the integral to spherical geometry. If the distance between the source of radiation and a field point is called $\vec{R} = \vec{R}_f - \vec{R}_s$, and \vec{n} is a unit vector in the direction of heat flow χ , the heat flux at the field point is given by

$$q = \int_{V_g} \frac{(4ke_g)(\vec{n} \cdot \vec{R})e^{-kR}}{(4\pi R^3)} dV$$

The following nondimensional variables can be defined:

$$\xi = \frac{e_g}{q_e}$$

$$\eta = \frac{q}{q_e}$$

$$\tau = k\chi$$

where χ is the independent variable.

The normalized heat flux can then be written as

$$\eta(\tau_f) = \int_{\tau_s} \xi(\tau_s) \int_s \frac{(\vec{n} \cdot \vec{R})e^{-kR}}{(\pi R^3)} ds d\tau_s \quad (48)$$

where s refers to surfaces of constant temperature and the subscripts f and s refer to field and source points for the radiation, respectively.

The equation for the heat flux is thus an integral equation of the first kind, with the kernel equal to

$$f(\tau_s, \tau_f) = \int_s \frac{(\vec{n} \cdot \vec{R})e^{-kR}}{(\pi R^3)} ds \quad (49)$$

This kernel is a very sharply peaked, discontinuous function when the optical thickness is large, which causes numerical difficulties when the equation is solved. In fact, it generally prevents obtaining results for opacities greater than about 10 or 20

The following numerical solution developed herein eliminated many of these difficulties. First, the nondimensional source strength was expanded in a Taylor series about the field point

$$\xi_s = \xi_f + (\tau_s - \tau_f)\xi'_f + \epsilon_{s,f}$$

Then, the following definitions were made

$$f_0(\tau_f) \equiv \int_{-\tau_e}^{+\tau_e} f(\tau_s, \tau_f) d\tau_s$$

$$f_1(\tau_f) \equiv \int_{-\tau_e}^{+\tau_e} f(\tau_s, \tau_f)(\tau_s - \tau_f) d\tau_s$$

The heat flux is then given by the following equation:

$$\eta_f = \xi_f f_{0,f} + \xi_f f_{1,f} + \int_{-\tau_e}^{+\tau_e} \epsilon_{s,f} f_{s,f} d\tau_s \quad (50)$$

An iterative solution to this equation can be found if $\epsilon_{s,f}$ and the edge boundary condition are those calculated on the previous iteration. Then, this equation becomes a first-order differential equation. Therefore, an integrating factor can be used to obtain the solution for the next iteration. If the following definitions are made,

$$g_f = \frac{\eta_f - \int_{-\tau_e}^{+\tau_e} \epsilon_{s,f} f_{s,f} d\tau_s}{f_{1,f}}$$

$$h_f = \frac{f_{0,f}}{f_{1,f}}$$

$$p_f = \int_0^{\tau_f} h_f d\tau$$

the solution is

$$\xi_f = e^{-p_f} \left(\xi_c + \int_0^{\tau_f} g_f e^{p_f} d\tau \right) \quad (51)$$

where

$$\xi_c = \xi_e e^{p_e} - \int_0^{\tau_e} g_e e^{p_e} d\tau$$

The first iteration is obtained from the diffusion approximation with jump boundary conditions. Convergence was assumed when the relative change in ξ from one iteration to the next was less than 0.0001. This numerical solution was programmed and used to obtain "exact" answers. The algebraic equations were compared to these exact answers.

RESULTS AND DISCUSSION

The major result of this study is equation (22), which has been derived in the analysis. In this section, the equation is used to display some characteristics of radiating gases. In the first subsection, some general features are presented. Then, the results of this study are compared with previously published information. Next, equation (22) is used to calculate center, edge, and average gas temperatures for opacities from 10^{-2} to 10^3 for slab, cylindrical, and spherical geometries. All these results are for a constant absorption coefficient. Lastly, a temperature distribution throughout a gas is calculated for a situation where the absorption coefficient varies with temperature and where the heat sources are not radially uniform. This calculation is presented to illustrate how this kind of relatively complicated problem can be handled.

General Characteristics

Some of the results are presented in terms of a brightness temperature T_b . This is the temperature of a blackbody (emissivity of 1) that is radiating the same heat flux as is leaving the gas under consideration. Thus, T_b is defined by the equation

$$q_e = \sigma T_b^4$$

Figure 3 affords a convenient way to relate absolute temperatures to actual heat fluxes. Since all the equations can be presented in terms of T/T_b , the use of this parameter is a convenient way to present curves that apply for any heat flux q_e .

Figure 4 depicts some typical temperature distributions of a radiating gas calculated with equation (8). The radial temperature distribution is shown for optical diameters of 0.01 (transparent), 1, 100, and 1000 (opaque). These curves are independent of the absolute, or dimensional, values of the heat flux, the gas diameter, and the absorption coefficient. Except for a region near the edge of an opaque gas, the gas temperatures are all higher than the brightness temperature. Also, the gas radiates most "efficiently" at some intermediate opacity. Efficient radiation means that the gas temperature is close to the brightness temperature. These results are for a sphere, but they are also typical of cylinders and slabs.

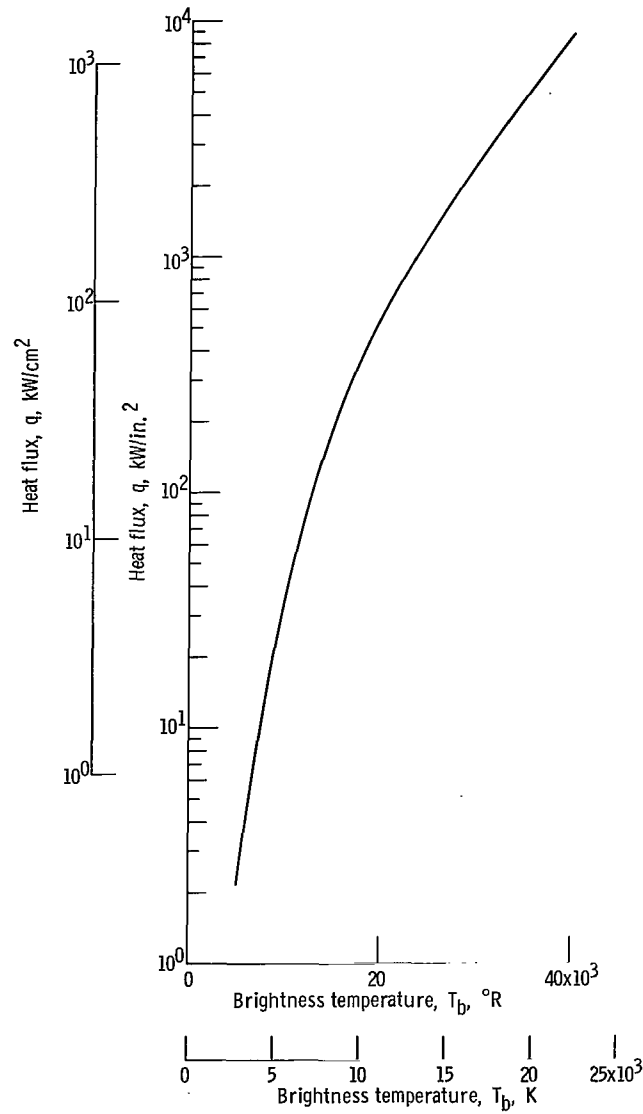


Figure 3. - Blackbody heat flux. Edge heat flux,
 $q_e = \sigma(T_b/10\,000)^4$; Stefan-Boltzmann constant,
 $\sigma = 34.8 \text{ kW}/\text{in.}^2/^\circ\text{R}^4$, $56.7 \text{ kW}/\text{cm}^2/\text{K}^4$, $33 \text{ Btu}/$
 $33 \text{ Btu}/(\text{sec})(\text{in.}^2)(^\circ\text{R}^4)$ ($\text{J}/(\text{sec})(\text{cm}^2)(\text{K}^4)$).

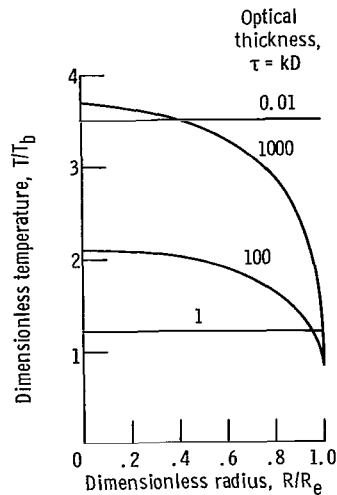


Figure 4. - Radial temperature profile in sphere; brightness temperature defined by $q_e = \sigma T_b^4$; curves calculated with equation (22).

Comparison With Other Results

The first comparison is made by examining the behavior of equation (22) in the two limiting cases of a transparent gas and an opaque gas. Consider a gas volume that is vanishingly small, so that its optical dimension kD goes to zero. If the gas is in thermodynamic equilibrium and has a refractive index of 1, the rate at which energy radiates from this volume is related to its temperature T_g by

$$Q = 4k\sigma T_g^4 \quad (52)$$

This assumes that the spectral energy distribution is given by the usual Planck function which when integrated over all wavelengths gives σT_g^4 .

Equation (52) gives the volumetric radiation from a gas at temperature T_g . For steady-state, this is also the volumetric heat source strength since the energy lost by radiation must be balanced by an equal energy gain. The heat flux leaving a gaseous region is related to the volumetric sources by the geometry, as follows:

$$\left. \begin{array}{ll} \text{Slab:} & q_e = \frac{QL}{2} \\ \text{Cylinder:} & q_e = \frac{QD}{4} \\ \text{Sphere:} & q_e = \frac{QD}{6} \end{array} \right\} \quad (53)$$

For a transparent gas (where the optical dimension is much less than 1), equation (52) gives the value of Q . Equations (52) and (53) give

$$\left. \begin{array}{ll} \text{Slab:} & q_e = (2\tau)\sigma T_g^4 \\ \text{Cylinder:} & q_e = (\tau)\sigma T_g^4 \\ \text{Sphere:} & q_e = \left(\frac{2\tau}{3}\right)\sigma T_g^4 \end{array} \right\} \quad (54)$$

These are "transparent gas" equations that relate the gas temperature to the radiated heat flux for small τ . Equation (22) gives these same equations for the limiting case where $\tau \rightarrow 0$. Therefore, equation (22) should give correct results for optically thin gases.

For the limiting case of an opaque gas, analysis (ref. 10) of radiation from stars has shown that the photosphere (or "edge") temperature is given by

$$T_e^4 = \frac{1}{2} T_b^4$$

Equation (22) produces this same result when $\tau \rightarrow \infty$ and $\chi = 1$ (the edge). Reference 10 also gives the exact answer as

$$T_e^4 = \frac{\sqrt{3}}{4} T_b^4$$

The star temperature distribution from the edge inward is given by reference 10 as

$$T^4 = \frac{1}{2} T_b^4 \left(1 + \frac{3\tau^*}{2}\right) \quad (55)$$

where τ^* is the depth into the star from the edge given in terms of photon mean free paths. Equation (22) gives this same form as $\tau \rightarrow \infty$, since $\tau^* = 1/2(1 - \bar{\chi})$ and $(1 - \bar{\chi}^2) = 2(1 - \bar{\chi})$ for $\bar{\chi}$ near 1.

Chandrasekhar (ref. 11) also presents a solution for temperatures near the edge of a star. It is obtained using his method of "successive approximations." The fourth approximation gives answers that are virtually exact. Figure 5 shows a comparison of temperatures obtained from equation (53), from Chandrasekhar (ref. 11), and from the computer solution for a sphere of optical diameter 80. The case chosen is for a brightness temperature of 10 000 K. The agreement is quite good. The general conclusion is that equation (22) is exact in the limit of a transparent gas, and sufficiently correct for an opaque gas for all practical purposes.

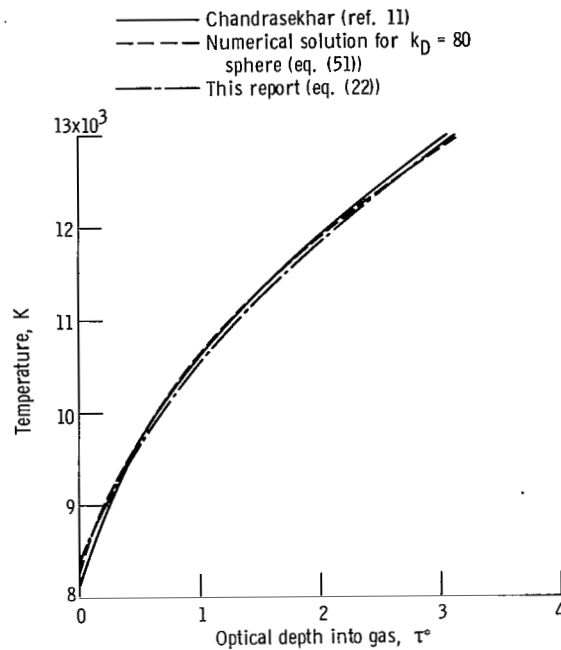


Figure 5. - Gas temperatures near edge for brightness temperature of 10 000 K.

Figure 6 shows a comparison of temperature distributions obtained with equation (22), with the jump boundary conditions of reference 12, and with the computer solution of the transport equation (eq. (51)). The comparison is for a slab geometry, and for a brightness temperature of 10 000 K. Both the jump boundary approximation and equation (22) are in close agreement with the numerical (exact) solution for optical thicknesses of 80 and 2.

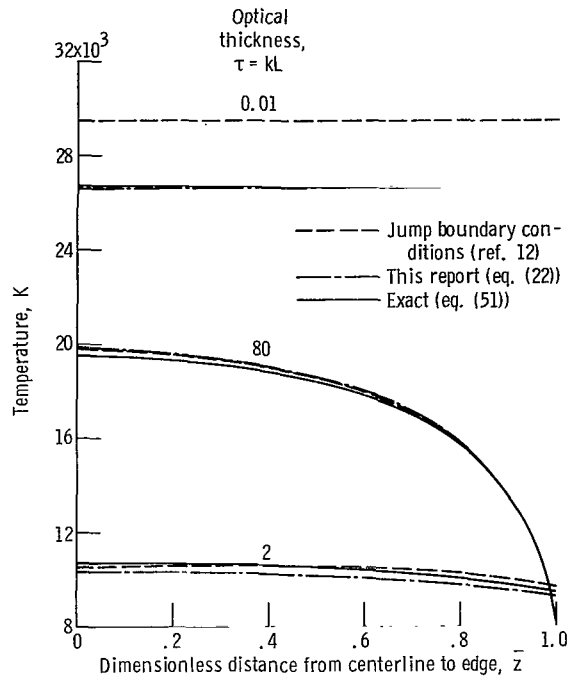


Figure 6. - Temperature distribution in slab for brightness temperature of 10 000 K.

For the optically thin case, equation (22) gives more accurate answers than are provided by jump boundary conditions. For a slab geometry, jump boundary conditions lead to errors of about 11 percent in temperature or about 50 percent in heat flux. For a cylinder, the corresponding numbers are 3 and 12 percent. For a sphere, jump boundary conditions give correct results.

For optically thick gases, equation (22) and jump boundary conditions give the same results. In the limiting case of $\tau \rightarrow \infty$, both methods overestimate edge temperature (for a given heat flux) by 3 percent or underestimate heat flux (for a given edge temperature) by 12 percent.

The idea of a gas edge emissivity is introduced at this point because it permits a convenient display and comparison of gaseous radiation characteristics over the entire range of opacities. This emissivity is based on the edge temperature. A blackbody at T_e would radiate

$$q_{b,e} = \sigma T_e^4$$

The actual heat flux radiated from the gas q will be different in most cases, and is given by

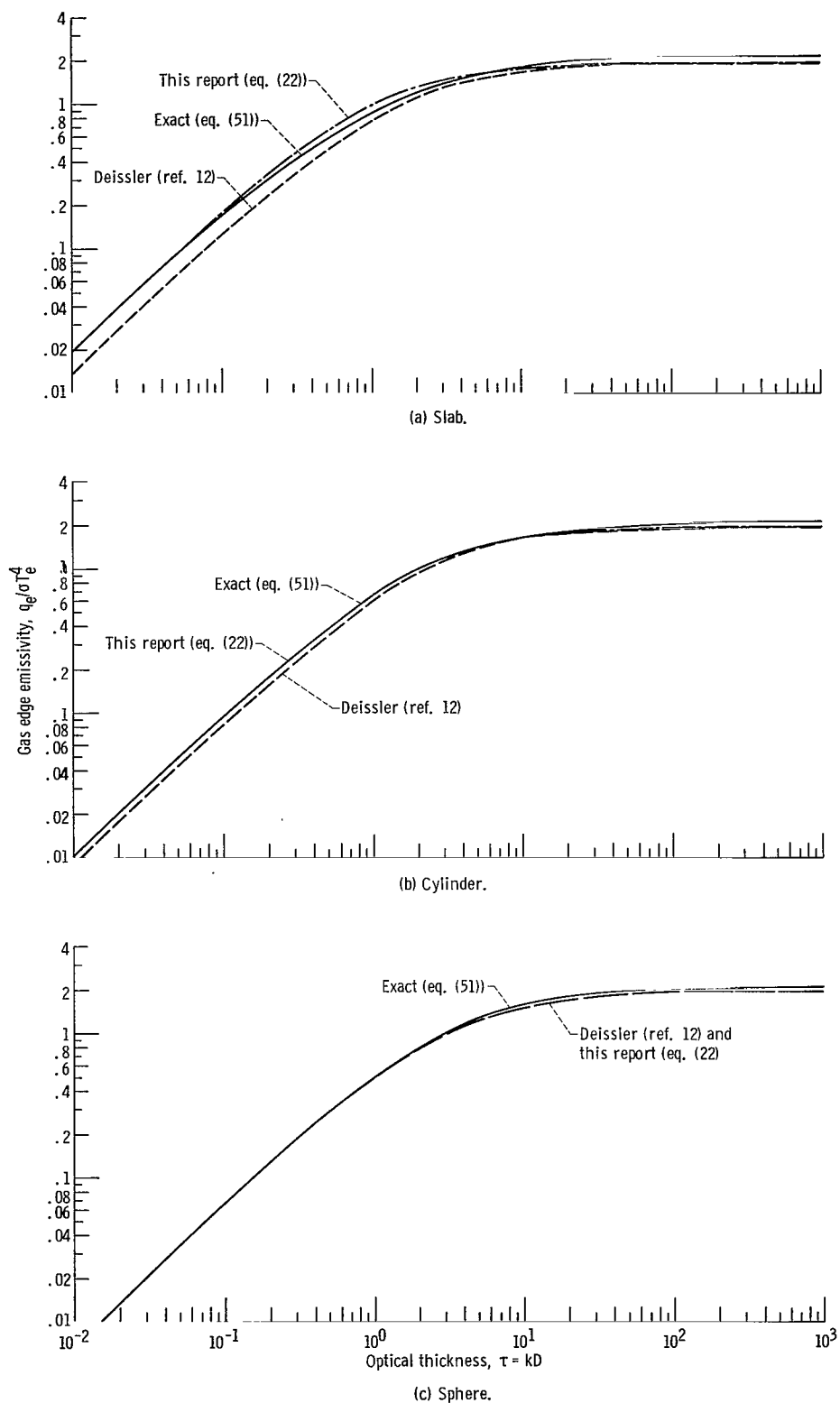


Figure 7. - Comparison of gas edge emissivity of slab, cylinder, and sphere based on edge temperature.

$$q_e = \epsilon \sigma T_e^4$$

This equation defines the gas edge emissivity. It is the ratio of radiated heat flux to the heat flux a blackbody at T_e would radiate.

Gas edge emissivity is shown in figure 7 for a slab, cylinder, and sphere, respectively. The curves labeled "this report" were calculated using equation (22); the constant a had the appropriate numerical value of 1, 2, or 3 as indicated in table I. The curves labeled "Deissler", were calculated using the constants 1.5, 2.25, or 3 (table I) that were obtained from jump boundary conditions. The curves labeled "exact" were obtained from the computer program described in the section ANALYSIS.

Gas emissivities are shown for an optical dimension τ from 10^{-2} to 10^3 . There is no particular significance to these limits, except that they include all important trends. Below an opacity of 10^{-2} , all of the curves continue to decrease without bound with a constant slope of 1. Above an opacity of 10^3 , all emissivities are essentially constant. The approximate equations give edge emissivities that approach the diffusion limit of 2. The computer solution appears to be approaching some higher value, in all probability the "exact" value of $4/\sqrt{3}$, or 2.31.

Figures 7(a) to (c) show that equation (22) gives somewhat better results than jump boundary conditions for opacities less than 1. Above $\tau = 1$, both methods are quite good. For a sphere, both approximations are good for all opacities. The jump boundary approach is least applicable (at $\tau < 1$) for a slab geometry. The reason for this geometry effect on the accuracy of jump boundary conditions is not apparent.

Figure 8 summarizes the edge emissivity of a radiating gas, as calculated from equation (22). It is interesting to note that the constant a in equation (22) is the value of

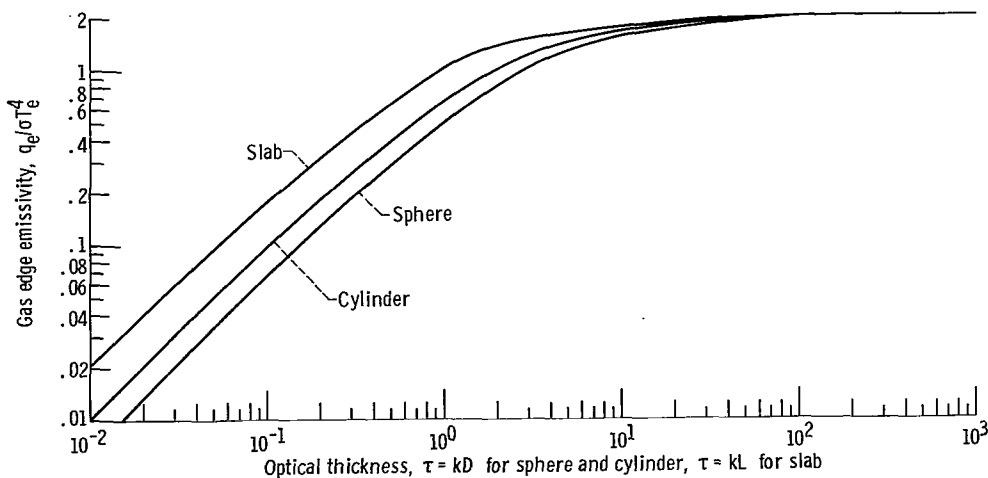


Figure 8. - Gas edge emissivity, based on edge temperature T_e , from equation (22).

τ that gives an emissivity of 1. That means that a is the value of τ for which the edge temperature is equal to the brightness temperature.

Calculation of Gas Temperatures

Figure 8 relates the radiated heat flux to the edge temperature. Equation (22) also yields internal gas temperatures. Figure 9 displays edge, average, and center temperatures as a function of optical dimension τ and geometry. The temperatures are normalized to the brightness temperature so that the curves apply for any heat flux.

The curves show the same general trends for all three geometries. Center and average temperatures are high for both transparent and opaque gases. Center and average temperatures have a minimum value that is within 10 percent of the brightness temperature. This minimum occurs at an optical dimension between 2 and 4. The edge temperature is high for a transparent gas, and decreases to an asymptotic value of 84 percent of the brightness temperature for τ greater than 10. For τ less than 1, the gas is at one constant temperature from edge to center.

Variable Absorption Coefficient and Nonuniform Heat Source Distribution

This section illustrates how to calculate temperatures in a radiating gas when the absorption coefficient varies with temperature and the heat sources are distributed non-uniformly in the gas. Equation (30) was derived for constant heat sources, but with an absorption coefficient given by

$$k = cT^n$$

An illustrative case is considered here where the heat source strength also varies throughout the gas.

The case to be considered is that of an induction-heated uranium gas. The problem is to estimate the temperature distribution in the gas. This problem is of current interest because induction heating is being used to study radiant-heat-transfer problems that would exist in a gaseous-uranium-fueled nuclear rocket engine (refs. 15 and 16).

It is first necessary to determine the mathematical relations that describe the variations of the absorption coefficient and the heat sources. Reference 17 has calculated uranium absorption coefficient as a function of temperature at 100 atmospheres. The case to be calculated here is at 1 atmosphere. The cross sections calculated at 100 atmospheres are assumed to hold at 1 atmosphere. Although this is not wholly true, it

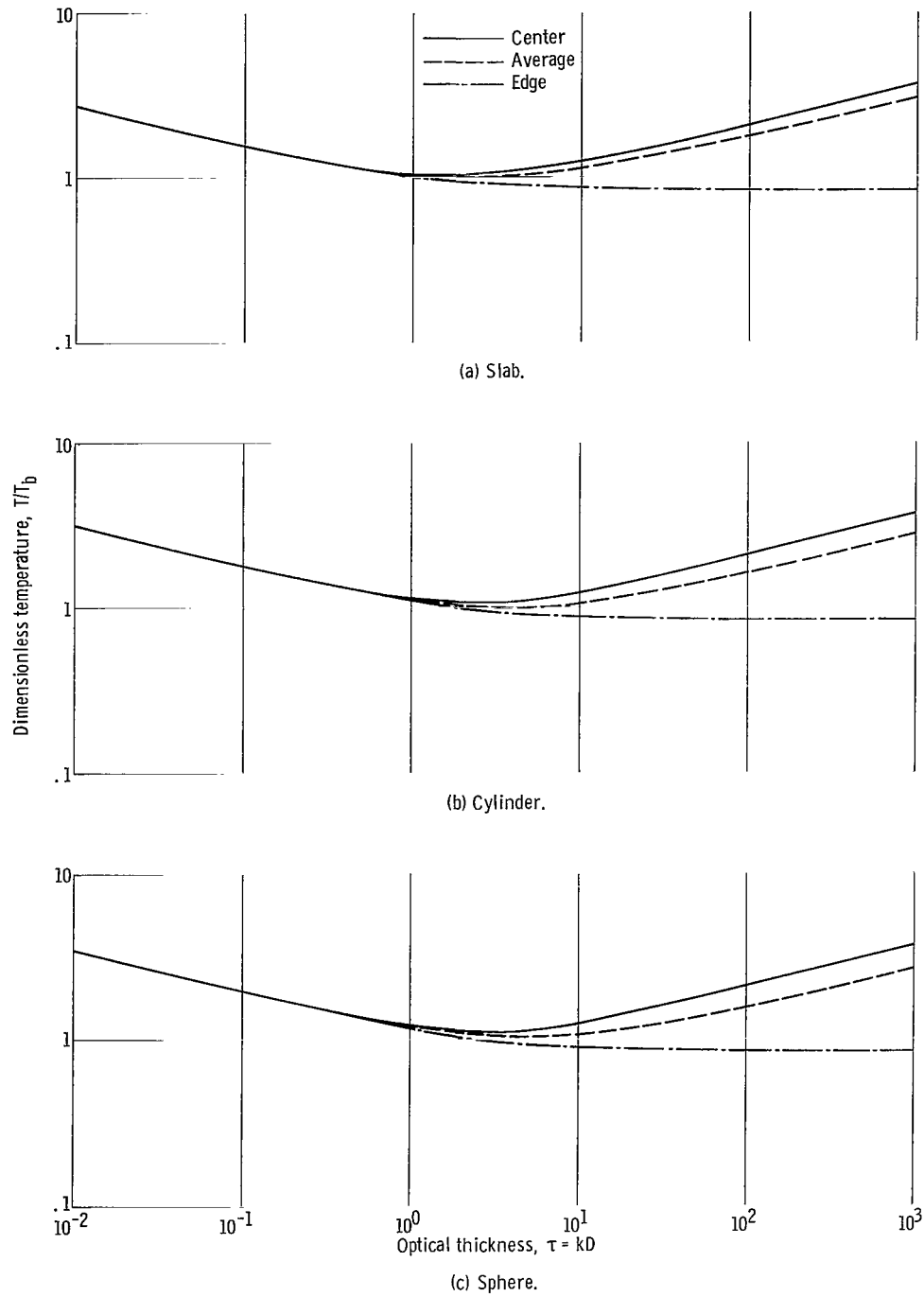


Figure 9. - Gas temperature of slab, cylinder, and sphere, normalized to brightness temperature T_b defined by $q_e = \sigma T_b^4$.

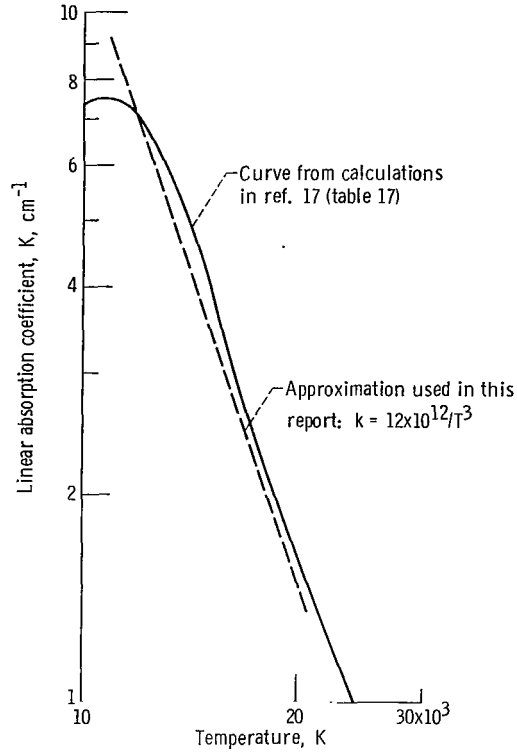


Figure 10. - Uranium absorption coefficient. Pressure, 1 atmosphere.

should be adequate for the purpose of this calculation. The curve obtained by dividing the values at 100 atmospheres (ref. 17) by 100 is shown in figure 10 as a solid curve.

Since a logarithmic straight line is more mathematically convenient, the true curve was approximated by the dashed line shown in figure 10. The equation of this line is

$$k = \frac{12 \times 10^{12}}{T^3}$$

where the absorption coefficient k is in inverse centimeters and T is in K.

For an induction-heated plasma, the spatial variation of the volumetric heating rate can be expressed in the form of a "decay factor":

$$\frac{Q(R)}{Q_e} = e^{y/\delta}$$

where $Q(R)$ and Q_e are the local heating rates at any radius R and at the edge radius

R_e , respectively; y is the distance radially inward from the edge; and δ is a skin depth (ref. 18). In order to obtain the skin depth, it is necessary to estimate the electrical resistivity. Following the method of reference 19 and using an electron density of 2.17×10^{17} per cubic centimeter yields the resistivity as 14×10^{-5} ohm-meter.

This finally results in a heat source distribution given by

$$\frac{Q}{Q_e} = e^{1.7(\bar{R}-1)}$$

and shown in figure 11. The dashed curve shows the parabolic approximation to this curve that was used in the example calculation because of mathematical convenience. The equation used is $Q/Q_e = c_1 + c_2 \bar{R}^2$ where $c_1 = 0.18$ and $c_2 = 0.82$.

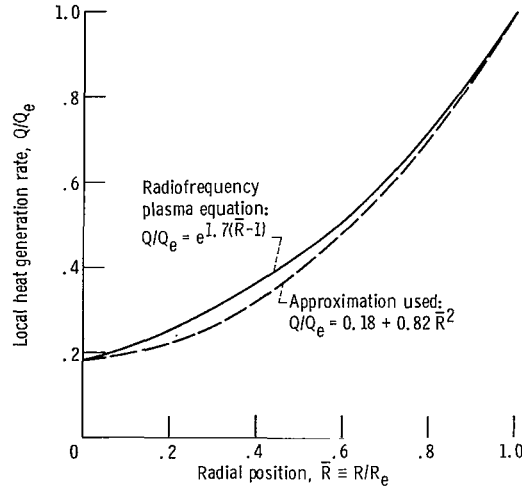


Figure 11. - Variable heat source distribution used in sample problem.

Carrying out the integration of this parabolic variation of Q for a cylinder, the temperature distribution is given by

$$\frac{T}{T_b} = \left\{ \frac{\frac{1}{2} + \frac{1}{\tau_e \left(c_1 + \frac{c_2}{2} \right)} + \left(\frac{4-n}{4} \right) \left(\frac{1}{c_1 + \frac{c_2}{2}} \right) \left(\frac{3\tau_e}{8} \right) \left[\frac{c_1}{2} (1 - \bar{R}^2) + \frac{c_2}{8} (1 - \bar{R}^4) \right]}{\left[\frac{1}{2} + \frac{1}{\tau_e \left(c_1 + \frac{c_2}{2} \right)} \right]^{n/4}} \right\}^{1/4-n}$$

The term $(c_1 + c_2/2)$ in this equation is (Q_{av}/Q_e) . If the heat source distribution is constant, then c_2 is zero and c_1 is 1, and this equation reduces to equation (30), which was for the case where $k = cT^n$. For a constant absorption coefficient, $n = 0$, and the equation further reduces to equation (22).

The foregoing equation may appear to be rather cumbersome, but it is quite easy to use. For the uranium plasma example, we choose a power of 500 kilowatts per centimeter of length, and a cylindrical plasma with a diameter and length of 1 centimeter. This gives a heat flux of 159 kilowatts per square centimeter, which corresponds to a brightness temperature of 12 900 K.

The edge temperature must be obtained first. The edge temperature is given by

$$\frac{T_e}{T_b} = \left[\frac{1}{2} + \frac{1}{\tau_e \left(c_1 + \frac{c_2}{2} \right)} \right]^{1/4}$$

There is an iteration required because τ_e is a function of T_e , since for this case

$$\tau_e = 2kR_e = 24 \times 10^{12} R_e T_e^{-3}$$

But the iteration quickly converges, with a good first guess at the edge temperature being the brightness temperature. The final answer for T_e is

$$T_e = 12\,200 \text{ K}$$

$$\tau_e = 6.6$$

The other values required are

$$c_1 = 0.18$$

$$c_2 = 0.82$$

$$n = -3$$

$$T_b = 12\,900 \text{ K}$$

These values result in the following equation for the temperature distribution in the uranium plasma:

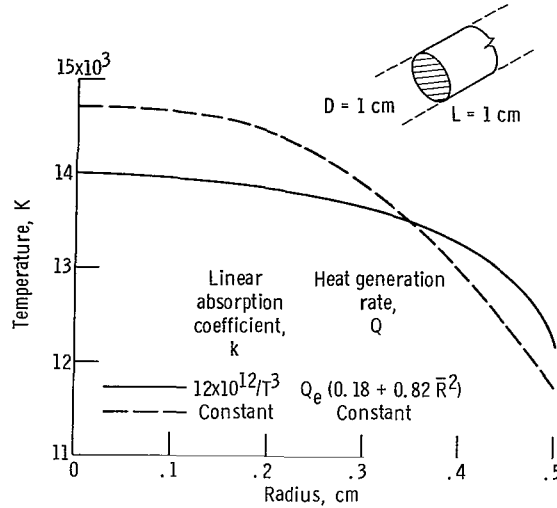


Figure 12. - Temperature distribution in radio frequency heated uranium plasma. Pressure, 1 atmosphere; power, 500 kilowatts per centimeter in length.

$$\frac{T}{12\,900} = \left[0.61 + 0.52(1 - \bar{R}^2) + 0.6(1 - \bar{R}^4) \right]^{1/7}$$

For the simple case of constant absorption coefficient ($\tau = 5.5$ is an average value for this case) and uniform heat generation, equation (22) gives

$$\frac{T}{12\,900} = \left[0.682 + 1.03(1 - \bar{R}^2) \right]^{1/4}$$

These two equations were used to obtain the temperature profiles shown in figure 12. It is apparent that, for the example case chosen, the combined effects of variable absorption coefficient and nonuniform heat generation do influence the temperature profile. The main point here is not the precise numbers obtained, but simply that a fairly complicated situation can be handled numerically. The conditions were deliberately chosen to incorporate:

- (1) Temperature-dependent absorption coefficient
- (2) Nonuniform heat generation
- (3) Intermediate optical dimension

Two restrictions remain. One is that the gas is gray. The other restriction is that diffusion approximation is applicable. For constant absorption coefficient and heat source distribution, the diffusion results are in good agreement with exact answers. This has not been shown for the case of variable absorption coefficient and heat source distribution. The mathematics have been carried out, but this does not assure correct answers.

Obviously, there may be combinations of a temperature-dependent absorption coefficient and nonuniform heat sources that cannot be handled by the diffusion approximation. To establish the validity of the variable absorption coefficient and heat source equations is a difficult problem that remains to be solved.

SUMMARY OF RESULTS

An analysis of radiant heat transfer from gray gases was carried out to obtain a simple, algebraic equation for the temperature distribution throughout the gas, from the edge to the center. The heat energy is generated by sources distributed within the gas. This heat is radiated to the surrounding environment, which is assumed not to radiate back. The analysis was for one-dimensional geometries; spheres, cylinders and slabs were considered.

Results are presented in terms of an optical dimension τ , which is the product of the absorption coefficient and the major dimension; that is, τ is given by kL for a slab of thickness L or by kD for a sphere or a cylinder of diameter D . Another generalizing parameter turned out to be a brightness radiating temperature T_b . This is the temperature of a blackbody (emissivity of 1) that would radiate the same heat flux as the gas.

For a constant absorption coefficient and uniformly distributed heat sources, the following results were obtained:

1. The edge temperature T_e is determined by the heat flux being radiated, the optical dimension τ , and the geometrical shape of the gas volume. It is given by

$$\frac{T_e}{T_b} = \left[\frac{1}{2} \left(1 + \frac{a}{\tau} \right) \right]^{1/4}$$

The constant a is 1, 2, or 3 for a slab, cylinder, or sphere, respectively.

2. The temperature distribution throughout the gas is given by

$$\frac{T}{T_b} = \left\{ \frac{1}{2} \left[1 + \frac{a}{\tau} + \frac{3\tau}{8} (1 - \bar{\chi}^2) \right] \right\}^{1/4}$$

where a is the same as in the edge temperature equation, and $\bar{\chi}$ is the dimensionless distance from the center of the gas to the edge.

3. These equations go to the proper asymptotic limits for a transparent gas and an opaque gas. At intermediate opacities (τ of 0.1 to 10), these equations give temperatures that are less than 3 percent different from answers obtained with a numerical solution of

the exact transport equation. Therefore, the equations are good approximations over the entire range of opacities.

The method of this report can also be numerically extended to the case of an absorption coefficient that is a function of temperature. For a flat plate geometry and an absorption coefficient given by $k = cT^n$, the temperature profile is

$$\frac{T}{T_b} = \left(\frac{\left\{ \frac{1}{2} \left[1 + \frac{1}{\tau_e} + \left(\frac{4-n}{4} \right) \left(\frac{3\tau_e}{8} \right) (1 - \bar{z}^2) \right] \right\}^{1/4-n}}{\left[\frac{1}{2} \left(1 + \frac{1}{\tau_e} \right) \right]^{n/4}} \right)$$

where \bar{z} is the dimensionless distance from the midplane to an edge surface. The optical thickness of the slab τ_e is based on the edge temperature. The edge temperature, and τ_e , are dependent only on the heat flux:

$$\frac{T_e}{T_b} = \left[\frac{1}{2} \left(1 + \frac{1}{\tau_e} \right) \right]^{1/4}$$

It is also possible to carry out the mathematics necessary to include the effect of a nonuniform distribution of heat sources. This is illustrated by deriving an equation for the temperature distribution in an induction-heated uranium plasma. In this situation, the absorption coefficient is a function of temperature, and the heat sources are a function of radial position in a cylindrical geometry. The results for variable absorption coefficient and heat sources were not compared with exact solutions.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 11, 1969,
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